

You should try to answer on your own before seeking H-E-L-P-S.

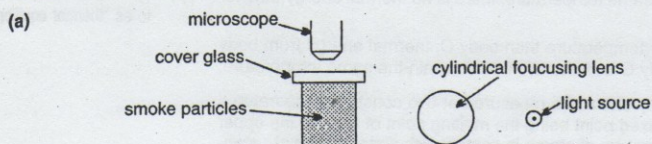
TOPIC opening

Question 1

- (a) Describe an experiment which demonstrates Brownian motion. [5]
 (b) Explain how Brownian motion provides evidence for the kinetic model of matter. [3]

[J99/P2/Q5]

Suggested Solution:



The apparatus used to demonstrate Brownian motion consists of a small transparent cell containing smoke particles, illuminated from the side by a light source. The smoke particles are observed through the microscope. A piece of cord is set smouldering and some of the smoke is collected by a syringe and transferred to the cell with the cover glass covering the cell. The microscope is then adjusted until some fine smoke particles can be viewed clearly as bright dots.

The particles can be observed to be continuously in random motion.

- (b) The random motion of the smoke particles cannot be due to any draft or convection currents. The motion of the smoke particles is due to the any movement of air molecules bombarding the smoke particles from all sides. The irregularity and incessant motion of the particles shows that air molecules move rapidly in all different directions all the time.

Question 2

- (a) State one similarity and one difference between conduction and convection of thermal energy. [2]
 (b) By reference to thermal energy transfer, explain what is meant by
 (i) two bodies having the same temperature.
 (ii) body H having a higher temperature than body C. [2]
 (c) (i) Briefly describe how a physical property may be used to measure temperature on its empirical centigrade scale.
 (ii) Hence explain why two thermometers measuring temperature on their empirical centigrade scales do not agree at all temperatures. [5]
 (d) Fig. 2.1 shows data for ethanol.

density	0.79 g cm ⁻³
specific heat capacity of liquid ethanol	2.4 J g ⁻¹ K ⁻¹
specific latent heat of fusion	110 J g ⁻¹
specific latent heat of vaporisation	840 J g ⁻¹
melting point	-120 °C
boiling point	78 °C

Fig. 2.1



Use the data in Fig. 2.1 to calculate the thermal energy required to convert 1.0 cm³ of ethanol at 20 °C into vapour at its normal boiling point. [6]

- (e) (i) State the *first law of thermodynamics*.
 (ii) Suggest why there is a considerable difference in magnitude between the specific latent heats of fusion and vaporisation. [5]

[J99/P3/Q2]

Suggested Solution:

(a) **Similarity:** Both modes of energy transfer require a medium.

Difference: In conduction, thermal energy is transferred through the vibration of molecules about their average positions which remain unchanged.

In convection, thermal energy is transferred through the bulk motion of a region in a fluid due to differences in the density arising from temperature differences.

(b) (i) When two bodies have the same temperature, there is no thermal energy transfer between them.

(ii) When body H has a higher temperature than body C, thermal energy from body H will be transferred to body C until the two bodies attain the same temperature.

(c) (i) Firstly, the physical property should be measured at two constant temperatures or fixed points - the lower fixed point being the melting point of ice and the upper fixed point being the temperature of steam in contact with water boiling at standard atmospheric pressure.

Secondly, a temperature on its empirical centigrade scale may be found by measuring the physical property and interpolating by using the following formula:

$$\theta = \frac{x_s - x_\theta}{x_s - x_i} \times 100 \text{ }^\circ\text{C}$$

where θ is a temperature being measured.

x_s is the value of the physical property at steam point.

x_i is the value of the physical property at ice point.

x_θ is the value of the physical property at the temperature θ .

(ii) The two thermometers do not agree at all temperatures because the physical properties used do not vary with temperature in exactly the same manner.

(d) Thermal energy required

= Thermal energy required to raise temperature of ethanol to its boiling point + Thermal energy required to vapourise the ethanol

$$= mc\Delta\theta + mL_v$$

$$= m(c\Delta\theta + L_v)$$

$$= pV(c\Delta\theta + L_v)$$

$$= (0.79 \text{ g cm}^{-3})(1.0 \text{ cm}^3)[2.4 \text{ J g}^{-1} \text{ K}^{-1}(78 - 20 \text{ K}) + 840 \text{ J g}^{-1}] = 774 \text{ J}$$

(e) (i) The First Law of Thermodynamics states that the total energy in a closed system is constant. Hence the increase in internal energy of a system is the sum of the work done on the system and the thermal energy supplied to the system.

(ii) There is no change in the temperature at fusion or vaporisation. Hence for both processes, the increase in internal energy, $\Delta U = 0$.

Hence $Q = -W$ i.e. thermal energy supplied to system = work done by system. The work done by the system during a phase change depends on the product of the external pressure, p , exerted on the system and the change in volume, ΔV , of the system.

The change in volume when ethanol changes from liquid into vapour is larger than when ethanol changes from solid to liquid. This is because, the ethanol molecules are spaced much more further apart in the vapour states compared to the liquid state. On the other hand, the spacing between the ethanol molecules in the solid and liquid states are similar. This means that work done by the ethanol in expanding against the atmosphere is larger during vaporisation than during melting. Hence, the thermal energy required for vaporisation is larger than that for melting.

(b) (i) This situation is referred to as "thermal equilibrium".

(d) The student need not convert all units for mass and dimension to kg and m, provided he is consistent in the substitution of the values as shown in the solution.



Question 3

Read the following passage.

Liquids, vapours and gases

When a small mass of water is introduced into a large evacuated vessel, the water evaporates completely to form what is called an unsaturated vapour. If the unsaturated vapour is then compressed slowly, the volume of the vapour decreases. During this change, the temperature is kept constant. This is known as an isothermal change and is shown by the line AB in Fig. 8.1.

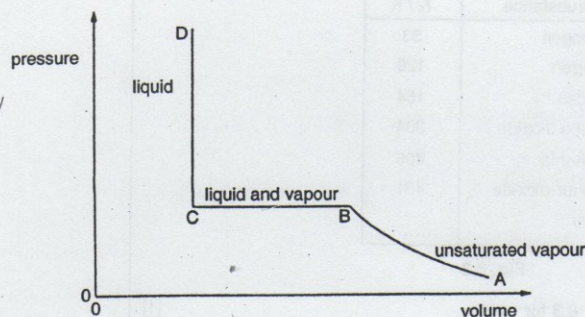


Fig. 8.1

On reducing the volume further at constant temperature, the vapour condenses and the pressure remains constant. In the region BC on Fig. 8.1, liquid and vapour are present and the vapour is said to be saturated. At C, all the vapour has condensed and only liquid is present. Any further reduction in volume is achieved only by applying very large pressures.

When values of the volume and the corresponding pressure are obtained at a number of different temperatures, a series of lines known as isotherms may be drawn as shown in Fig. 8.2. Each isotherm is drawn for the same mass of water.

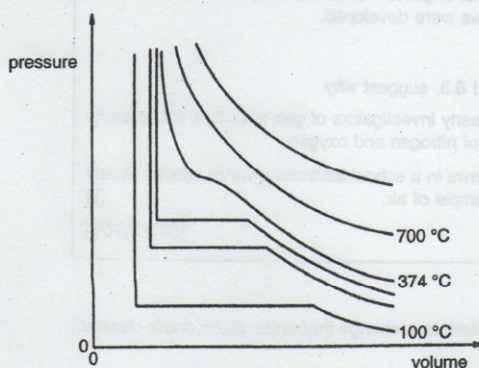


Fig. 8.2

Answer the following questions.

(a) By reference to the passage,

(i) state what is meant by

1. 'an isothermal change',
2. 'the vapour condenses'.

[3]

(ii) distinguish between an *unsaturated* vapour and a *saturated* vapour.

[2]

- (iii) distinguish between the behaviour of an unsaturated vapour and a saturated vapour when the volume is reduced at constant temperature. [2]
- (b) State the feature of the line on Fig. 8.1 which indicates that large changes in pressure are required to produce small changes in volume of a liquid. [1]
- (c) In order to produce liquid by increasing the pressure alone, the vapour must be below a particular temperature which is different for different substances. This temperature is known as the critical temperature T_c of the substance. Fig. 8.3 lists some substances and the corresponding critical temperatures, measured in kelvin.

substance	T_c / K
hydrogen	33
nitrogen	126
oxygen	154
carbon dioxide	304
ammonia	406
sulphur dioxide	431
water

Fig. 8.3

- Use Fig. 8.2 complete Fig. 8.3 for water. [1]
- (d) In early experiments to try to liquefy gases, increase in pressure alone was used. Gases which could not be liquefied at room temperature by pressure alone were known as *permanent gases*.
- (i) List the substances in Fig. 8.3 which would have been known as permanent gases.
- (ii) Suggest, with a reason, which substance listed in Fig. 8.3 proved to be most useful in early experimental work on the behaviour of a vapour near to its critical temperature.
- (e) Scientists investigating the behaviour of gases first used atmospheric air. As a result of their investigations, the gas laws were developed.
- (i) State the ideal gas equation.
- (ii) By reference to Figs 8.2 and 8.3, suggest why
- it was fortunate for the early investigators of gas laws that atmospheric air is composed mainly of nitrogen and oxygen.
 - when verifying the gas laws in a school laboratory, water vapour should be removed from the sample of air. [5]

[D99/O2/Q8]

Suggested Solution:

- (a) (i) 1. 'An isothermal change' refers to a change that takes place under constant temperature.
2. 'The vapour condenses' refers to the phenomenon where the water undergoes a change of state from the gaseous to the liquid state.
- (ii) An unsaturated vapour does not coexist with any liquid but a saturated vapour does.
- (iii) The pressure exerted by an unsaturated vapour increases when the volume is reduced at constant temperature but the pressure exerted by a saturated vapour remains constant when the volume is reduced at constant temperature.
- (b) Segment DC of the line on Fig. 8.1 shows that large changes in pressure are required to produce small changes in volume of a liquid, as shown by the large gradient of the line.
- (a) (ii) A saturated vapour is one that is in contact with its own liquid.



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(c)

substance	T_c / K
hydrogen	33
nitrogen	126
oxygen	154
carbon dioxide	304
ammonia	406
sulphur dioxide	431
water	647

- (d) (i) Carbon dioxide
Ammonia
Sulphur dioxide, water

- (ii) Carbon dioxide could be the most useful substance in early experimental work on the behaviour of a vapour near to its critical point. Its critical point is just less than 10 K above typical room temperature which may be obtained easily. Oxygen and ammonia, for instance, have critical temperatures that are very far from typical room temperatures which means that much energy is needed to cool the gas down (in the case of oxygen) or to heat up the gas (in the case of ammonia) in order to bring the gas to its critical temperature.

(e) (i)
$$pV = nRT$$

where P : pressure exerted by gas.

V : volume occupied by gas.

n : number of moles of gas.

R : molar gas constant.

T : temperature.

- (ii) 1. From Fig. 8.2, it may be seen that the ideal gas equation holds only for temperatures that are much higher than the critical temperature of the gas. For example, the isotherms for temperatures above 700 °C Boyle's law ($pV = \text{constant}$) as seen in the shape of the isotherms.

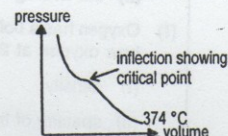
From Fig. 8.3, it may be seen that nitrogen and oxygen have critical temperatures that are much lower than typical room temperatures. Early investigators of gas laws experiment with temperature ranges around room temperature. At these temperatures, nitrogen and oxygen, being at temperatures much higher than their critical temperatures, act as ideal gases. These might have made it easier for early investigators using atmospheric air, to deduce the ideal gas equation from their results.

2. From Fig. 8.3, it may be seen that water vapour may be liquefied by increasing pressure alone at room temperatures. If water vapour is not removed from the sample of air used to verify gas laws, the water vapour present will be liquefied when large pressures are applied. From Fig. 8.2, it may be seen that the ideal gas equation does not hold where water vapour and its liquid form coexist.

- (c) The isotherm for 374 °C does not show an extended plateau but shows a point of inflection. This shows that the critical temperature of water is 374 °C. The temperature in Kelvin is

$$374 + 273.15 = 647 \text{ K}$$

(correct to 3 S.F.)



- (d) (i) Room temperatures are typically about 25 °C i.e. $273.15 + 25 = 298 \text{ K}$.

In order to produce liquid by increasing the pressure alone, the vapour must be below its critical temperature. Permanent gases are those that could not be liquefied at room temperature by pressure alone. This means that permanent gases could be those that have critical temperatures above room temperature ($\sim 298 \text{ K}$).

- (ii) Andrews made experiments on carbon dioxide in 1869.

Question 4

- (a) Define the term *density*. [1]
- (b) Outline how molecular movement causes the pressure exerted by a gas. [2]
- (c) One mole of oxygen has a mass of 32 g. Assuming oxygen behaves as an ideal gas, calculate
- (i) the volume occupied by one mole of oxygen gas when at temperature 273 K and pressure $1.01 \times 10^5 \text{ Pa}$. [1]
- (ii) the density of oxygen gas at this temperature and pressure. [5]





- (d) (i) Explain what is meant by the root-mean-square speed $\sqrt{\langle c^2 \rangle}$ of gas molecules.
- (ii) Calculate the root-mean-square speed of four molecules travelling with speeds 300 ms^{-1} , 400 ms^{-1} , 500 ms^{-1} and 600 ms^{-1} . [4]
- (e) Assuming ideal gas behaviour, calculate for oxygen at 273 K
- (i) the root-mean-square speed of its molecules.
- (ii) the average kinetic energy of a molecule. [4]
- (f) Oxygen has a boiling point of 90 K and a melting point of 55 K. Describe qualitatively how oxygen at 273 K and oxygen at 27 K differ in respect of
- (i) density,
- (ii) spacing of the molecules,
- (iii) order in the pattern of molecules,
- (iv) motion of the molecules. [4]

[D99/P3/Q4]

Suggested Solution:

- (a) The term 'density' refers to the mass per unit volume.
- (b) The molecules in a gas are in constant and random motion. As the molecules bombard against the walls of the container containing the gas, the molecules exert force on the container walls.

The force exerted per unit area of the container walls is the pressure exerted by the gas.

(c) (i) $pV = nRT$
 $V = \frac{nRT}{p} = \frac{1 \times 8.31 \times 273}{1.01 \times 10^5} = 2.25 \times 10^{-2} \text{ m}^3$

(ii) $\rho = \frac{M}{V} = \frac{32 \times 10^{-3}}{2.25 \times 10^{-2}} = 1.42 \text{ kg m}^{-3}$

- (d) (i) The root-mean-square speed of gas molecules refers to the square root of the average of the squared speeds of the gas molecules.

(ii) Root-mean-square speed of the four molecules = $\sqrt{\frac{300^2 + 400^2 + 500^2 + 600^2}{4}}$
 $= 464 \text{ ms}^{-1}$

- (e) (i) For ideal gas,

$$\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.31)(273)}{32 \times 10^{-3}}} = 461 \text{ ms}^{-1}$$

(ii) Average kinetic energy of a molecule = $\frac{3}{2}kT$
 $= \frac{3}{2}(1.38 \times 10^{-23})(273) = 5.65 \times 10^{-21} \text{ J}$

- (f) (i) The density of oxygen at 273 K is lower than the density of oxygen at 27 K.
- (ii) The spacing of the molecules in oxygen at 273 K is greater than that in oxygen at 27 K.
- (iii) There is no fixed order in the pattern of molecules in oxygen at 273 K but the molecules in oxygen at 27 K are ordered in a particular pattern.
- (iv) The molecules in oxygen at 273 K are in constant random motion but the molecules in oxygen at 27 K vibrate about their mean positions.
- (f) Since the boiling point of oxygen is 90 K, it is in its gaseous state at 273 K. Since the melting point of oxygen is 55 K, it is in its solid state at 27 K.



**Question 5**

- (a) (i) Distinguish between the processes of evaporation and of boiling.
 (ii) Use the first law of thermodynamics to explain why, when a liquid evaporates or boils, thermal energy must be supplied to the liquid in order to maintain constant temperature. [6]
- (b) The pressure p of an ideal gas of density ρ is related to the mean-square speed $\langle c^2 \rangle$ of its molecules by the expression

$$p = \frac{1}{3} \rho \langle c^2 \rangle.$$

- (i) State three basic assumptions of the kinetic theory of gases, which lead to a model of an *ideal* gas. [3]
 (ii) Write down the equation of state for an ideal gas. [1]
 (iii) Show that the average kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature T . [4]
- (c) Free neutrons in the core of a fission reactor are sometimes referred to as a 'neutron gas'. These free (thermal) neutrons may be assumed to behave as molecules of an ideal gas at a temperature of 35 °C.
- (i) Calculate, for a free neutron of mass 1.67×10^{-27} kg.
 1. Its mean kinetic energy.
 2. its root-mean-square (r.m.s.) speed.
- (ii) Determine the temperature of helium gas, assumed to be an ideal gas, at which helium molecules (each of mass $4u$) would have the same r.m.s. speed as the free neutrons. [6]

[J00/P3/Q5]

Suggested Solution:

- (a) (i) Evaporation occurs at any temperature but boiling occurs only at the boiling point.
 (ii) Molecules overcome the attractive forces exerted by surrounding molecules and escape from the liquid surface when a liquid evaporates or boils. In so doing, the liquid is actually doing work against the atmosphere. According to the first law of thermodynamics, internal energy remains constant when the work done by a system equals the thermal energy supplied to it. Hence to maintain constant temperature (implying constant internal energy), thermal energy must be supplied to a liquid which is evaporating or boiling.
- (b) (i) Three basic assumptions:
 1. A gas consists of particles called molecules.
 2. The molecules are in constant random motion.
 3. The range of intermolecular forces is small compared to the average separation of the gas molecules such that intermolecular forces are negligible except during a collision, the duration of a collision is negligible compared to the time spent in free motion and the volume of the gas molecules is negligible compared to the volume occupied by the gas.

$$(ii) \quad pV = nRT$$

where V : volume of the gas
 n : number of moles of the gas
 R : molar gas constant
 T : temperature of the gas

- (a) (ii) First law of thermodynamics:

$$\begin{aligned} \Delta U &= Q + W \\ \Delta U &= 0 \text{ when } Q + W = 0 \\ \Rightarrow Q &= -W \end{aligned}$$

This means that the work done by the liquid while it evaporates or boils ($-W$) must equal the thermal energy supplied (Q) in order for temperature to be constant since internal energy (U) is proportional to temperature.

- (b) (i) Other assumptions:
 4. A molecule moves with uniform velocity between collisions.
 5. The collisions of the molecules with one another and with the walls are perfectly elastic on average.
 6. Newtonian mechanics can be applied to the molecular collisions.





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$$(iii) \rho = \frac{1}{3} \rho \langle c^2 \rangle \text{ and } pV = nRT$$

$$\Rightarrow \frac{1}{3} \rho \langle c^2 \rangle = \frac{nRT}{V}$$

$$\frac{1}{3} \frac{M}{V} \langle c^2 \rangle = \frac{nRT}{V} \text{ where } M : \text{mass of gas}$$

$$\frac{1}{3} n N_A m \langle c^2 \rangle = nRT \text{ where } m = \text{mass of a molecule}$$

$$m \langle c^2 \rangle = \frac{3RT}{N_A}$$

$$\Rightarrow \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT \text{ where } k : \text{Boltzmann's constant} = \frac{R}{N_A}$$

Hence the average kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature T.

$$(c) (i) 1. \text{ Mean kinetic energy of a free neutron} = \frac{1}{2} m \langle c^2 \rangle$$

$$= \frac{3kT}{2}$$

$$= \frac{3(1.38 \times 10^{-23})(273 + 35)}{2}$$

$$= 6.4 \times 10^{-21} \text{ J}$$

$$2. \text{ Root-mean-square (r.m.s.) speed} = \sqrt{\langle c^2 \rangle}$$

$$= \sqrt{\frac{\text{mean kinetic energy} \times 2}{\text{mass of neutron}}}$$

$$= \sqrt{\frac{6.4 \times 10^{-21} \times 2}{1.67 \times 10^{-27}}}$$

$$= 2768.5 \approx 2770 \text{ ms}^{-1} \text{ (3s.f.)}$$

$$(ii) \frac{1}{2} m \langle c^2 \rangle = \frac{3}{2} kT$$

$$T = \frac{m \langle c^2 \rangle}{3k} = \frac{4u \langle c^2 \rangle}{3k} = \frac{4(1.66 \times 10^{-27})(2768.5^2)}{3(1.38 \times 10^{-23})} = 1229 \text{ K}$$

Question 6

(a) (i) State what is meant by the *internal energy* of a system.

(ii) Explain why the internal energy of an ideal gas is proportional to the mean-square speed $\langle c^2 \rangle$ of its atoms. [4]

(b) The first law of thermodynamics may be expressed as

$$\text{gain in internal energy} = q + w.$$

Explain the symbol

(i) q .

(ii) w . [3]

[D00/P2/Q7]

Suggested Solution:

(a) (i) The internal energy of a system is the sum of all the microscopic kinetic and potential energies of the molecules within the system.

(ii) The attraction between the atoms in an ideal gas is negligible. Hence, the potential energies of the atoms are negligible. This means that the internal energy of an ideal gas is just the sum of all the kinetic energies of the atoms. Since the kinetic energies are proportional to the mean-square speed of its atoms, the internal energy of the ideal gas is also proportional to the mean-square speed of its atoms.

(b) (i) Heat supplied to the system

(ii) Work done on the system



Question 7

- (a) (i) Define *linear momentum*.
 (ii) Use your definition of momentum to define *force*.
 (iii) Show that this definition leads to the equation $F = m \times a$.
 (iv) State the principle of conservation of momentum. [7]

- (b) In a gas a hydrogen molecule, mass 2.00 u and velocity $1.88 \times 10^3 \text{ ms}^{-1}$, collides elastically and head-on with an oxygen molecule, mass 32.0 u and velocity 405 ms^{-1} , as illustrated in Fig. 1.1.

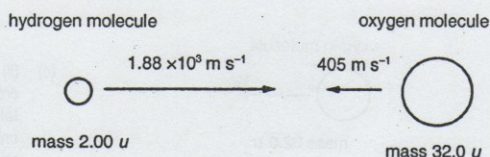


Fig. 1.1

In qualitative terms, what can be stated about the subsequent motion as a result of knowing that

- (i) the collision is elastic,
 (ii) the collision is head on? [3]
- (c) Using your answers to (b),
 (i) determine the velocity of separation of the two molecules after the collision,
 (ii) apply the law of conservation of momentum to the collision,
 (iii) determine the velocity of both molecules after the collision. [6]
- (d) The equation $T = m\langle c^2 \rangle / 3k$ gives the temperature T of molecules in terms of their mass m , their mean square speed $\langle c^2 \rangle$ and the Boltzmann constant k . Using this equation for the oxygen and hydrogen molecules before the collision in (b), gives a value of T for the hydrogen molecule of 283 K , and for the oxygen molecule of 210 K .
 (i) Explain why it is difficult to justify the application of this equation to this situation.
 (ii) How is it possible for the two molecules in the gas to have different values of T before the collision? [4]

[D00/P3/Q1]

Suggested Solution:

- (a) (i) *Linear momentum* is the product of the mass of a body and its velocity.
 (ii) A *force* is a push or a pull that, when it acts on an object, tends to cause the momentum of the object to change.
 (iii) According to Newton's second law, the force acting on an object is proportional to its rate of change of its momentum. Hence

$$F \propto \frac{d(mv)}{dt}$$

$$F = km \frac{dv}{dt}$$

where k is the constant of proportionality. In SI units, k is taken as one.

$\therefore F = m \times a$ since acceleration is the rate of change of velocity.



- (iv) The Principle of Conservation of Momentum states that in any system in which no external forces act, the total momentum of the objects in the system in any given direction remains constant.
- (b) (i) The total kinetic energies of the molecules after the collision is the same as it was before the collision.
- (ii) The two molecules move off in opposite directions.
- (c) (i) Since the two molecules collide head-on and move off in opposite directions, the velocity of separation of the two molecules after the collision equals the relative velocity with which they approach before collision

$$= 1.88 \times 10^3 + 405$$

$$= 2.29 \times 10^3 \text{ ms}^{-1}$$

- (ii) After the collision:



By the Law of Conservation of Momentum,

$$\text{Total momentum after collision} = \text{Total momentum before collision}$$

$$(2.00u)(-v_H) + (32.0u)(v_O) = (2.00u)(1.88 \times 10^3) + (32.0u)(-405)$$

$$32v_O - 2v_H = -9200$$

$$\text{i.e. } v_H = 4600 + 16v_O$$

- (iii) From part (c)(i),

$$v_O = (-v_H) = 2.29 \times 10^3$$

$$\text{i.e. } v_O + v_H = 2.29 \times 10^3$$

From part (c)(ii),

$$v_O + (4600 + 16v_O) = 2.29 \times 10^3$$

$$v_O = \frac{2.29 \times 10^3 - 4600}{17} = -136 \text{ ms}^{-1}$$

Hence, the velocity of the oxygen molecule after the collision is 136 ms^{-1} directed to the left.

$$\text{Now, } v_H = 4600 + 16(-136)$$

$$= 2.43 \times 10^3 \text{ ms}^{-1}$$

- (d) (i) It is difficult to justify the application of this equation to this situation because the equation is more applicable in a system containing a large number of molecules of varying speed in random motion.
- (ii) It is possible for the two molecules in the gas to have different values of T before the collision by confining them in thermally insulated and separate containers and then allowing them to mix through diffusion.

- (c) (ii) To start solving this problem, sketch a diagram, labelling the known and unknown quantities. Make a guess of the directions of v_H and v_O and indicate them on the diagram.

Since there are two unknown quantities, v_O and v_H , form a pair of simultaneous equations and solve them. The convention of taking vectors directed to the right as positive is taken.

(iii) The negative sign for v_O simply means that the direction of v_O is opposite to the supposed direction that was taken while solving this problem.

Question 8

- (a) Derive from the definitions of density and pressure, the equation $p = \rho gh$ for the pressure exerted by a column of liquid of height h and density ρ . [3]
- (b) (i) Using the kinetic model of a gas, explain how a pressure is exerted by a gas.
- (ii) Calculate the root-mean-square speed of gas molecules in a gas at pressure $1.05 \times 10^5 \text{ Pa}$ and of density 1.29 kg m^{-3} . [5]
- (c) (i) State two physical properties which may be used for the measurement of temperatures. [2]





- (ii) Describe the principle features of a thermocouple thermometer and indicate its advantage and disadvantages. [5]
- (iii) An ideal gas is defined as one for which, at constant pressure, the volume of the gas is proportional to the absolute temperature. Calculate the absolute temperature T when an ideal gas has volume 0.00783 m^3 , assuming that the same mass of the ideal gas had volume 0.00308 m^3 when at the same pressure and at temperature 273 K . [2]
- (iv) Express 273 K and your value of T from (iii) as temperatures measured on the Celsius scale. [1]
- (v) Comment on the statement 'Today the temperature is 40°C and yesterday it was 20°C so it is twice as hot today as it was yesterday'. [2]

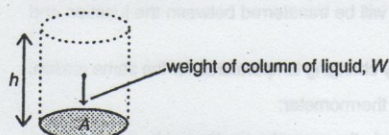
[D00/P3/Q5]

Suggested Solution:

- (a) Density is defined as the mass per unit volume, i.e. $\rho = \frac{m}{V}$.

Pressure is defined as the force per unit area, $p = \frac{F}{A}$.

Consider a horizontal area A at a depth h below the surface of the liquid.



Force, F acting on the area A = Weight, W of the column of liquid

$$\begin{aligned} F &= mg \\ &= \rho Vg \quad (\because \rho = \frac{m}{V}) \\ &= \rho Ahg \quad (\because V = Ah) \end{aligned}$$

Hence the pressure, p acting on the area A is

$$p = \frac{F}{A} = \frac{\rho Ahg}{A} = \rho gh$$

- (b) (i) In the kinetic model, a gas consists of a great number of molecules, behaving like perfectly elastic spheres, in random motion. Whenever a molecule bounces off a wall of the container, the force which it exerts on the wall equals the rate of change of its momentum. The average force exerted by the gas on the whole container is the average rate at which the momentum of the molecules changes due to collision with the walls. The pressure exerted by the gas on the container walls is hence the force per unit area of the walls.

(ii) $p = \frac{1}{3} \rho \langle c^2 \rangle$

Root-mean-square speed of the gas molecules,

$$\sqrt{\langle c^2 \rangle} = \sqrt{\frac{3p}{\rho}} = \sqrt{\frac{3 \times 1.05 \times 10^5}{1.29}} = 494 \text{ ms}^{-1}$$

- (c) (i) Two physical properties which may be used for the measurement of temperature:

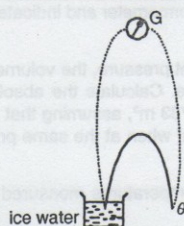
- length of a liquid column.
- electrical resistance of a metal.

- (c) (i) Other physical properties:

- volume of a gas.
- e.m.f. of the junction formed by two different metals.



(ii)



A simple thermocouple thermometer consists of wires of different materials, such as iron and copper, welded together to form junctions. One junction is placed in melting ice which provides the reference temperature while the other junction is placed in contact with the object at unknown temperature θ . A sensitive galvanometer G is used to measure the e.m.f., E arising from the difference in temperatures of the two junctions. The unknown temperature θ can then be derived from the relation

$$E = a + b\theta + c\theta^2$$

where a , b , c are constants determined by measuring E at known fixed points.

Advantages of the thermocouple thermometer:

1. The fine junction can be used to accurately measure the temperature of small objects because very little heat will be transferred between the junction and the object.
2. It can be used to measure rapidly changing temperatures for the same reason.

Disadvantages of the thermocouple thermometer:

1. The accuracy of the thermocouple thermometer is affected by the presence of small differences in purity of the metals forming the junctions.
2. The accuracy of the thermometer is also affected by small differences of temperature e.g. due to the warmth of the hand.

(iii) At constant pressure,

$$V \propto T \Rightarrow \frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\Rightarrow T_2 = \frac{V_2}{V_1} \times T_1 = \frac{0.00783}{0.00308} \times 273 = 694 \text{ K}$$

(iv) $\theta = T - 273.15 \text{ K}$

where θ represents the temperature on the Celsius scale.

T represents the temperature on the absolute scale.

For $T = 273 \text{ K}$,

$$\theta = 273 - 273.15 = -0.15 \text{ }^\circ\text{C}$$

For $T = 694 \text{ K}$,

$$\theta = 694 - 273.15 = 421 \text{ }^\circ\text{C} \text{ (3 significant figures)}$$

(v) The statement is made upon the assumption that the human perception of hotness is directly proportional to the temperature with $0 \text{ }^\circ\text{C}$ as the reference point. However, this may not be so because the perception of hotness is not absolutely the same for different people. Moreover, it is a subjective perception that is difficult to quantify; a temperature reading gives an indication of how high or low the degree of hotness is but it may not be appropriate to use the readings in this way. In addition, different thermometers using different physical properties would give different temperature readings.

**Question 9**

- (a) State three of the assumptions of the kinetic theory of gases. [3]
- (b) Consider a single molecule of mass m in a cubical container of internal side length l . The molecule is travelling with velocity v directly towards one of the walls W as shown in Fig. 3.1. Collisions between the molecule and the walls are elastic.

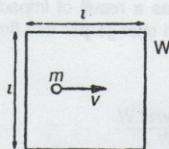


Fig. 3.1

Deduce

- (i) the change in the molecule's momentum when it collides with wall W ,
- (ii) the time between collisions with wall W .
- (iii) the momentum change per unit time for the molecule at wall W ,
- (iv) the average force on wall W as a result of impacts by the molecule,
- (v) the average pressure on wall W . [6]
- (c) The pressure p of an ideal gas is given by the equation

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

Explain briefly how this result can be deduced by modifying the equation derived in (b)(v). [3]

- (d) Use the equation of state for an ideal gas, together with the equation given in (c), to show that the average translational kinetic energy of a molecule is proportional to the absolute temperature T . [3]
- (e) A container of gas holds 3.6×10^{25} molecules of an ideal gas each with a mass of 4.6×10^{-26} kg. The root-mean-square speed of the molecules is 270 ms^{-1} and the container is on an aircraft travelling at 240 ms^{-1} . Calculate
- (i) the kinetic energy as a result of the random motion of all the molecules in the gas.
- (ii) the kinetic energy the gas has as a result of being on the aircraft,
- (iii) the internal energy of the gas. [5]

[D02/P3/Q3]

Suggested Solution:

- (a) Three of the assumptions of the Kinetic Theory of Gases:

- (1) A gas consists of particles called molecules.
- (2) A molecule moves with uniform velocity between collisions.
- (3) The collisions of the molecules with one another and with the walls are perfectly elastic.

- (b) (i) Change in the molecule's momentum when it collides with wall W
 = mass \times (final velocity - initial velocity)
 = $m(v - (-v)) = 2mv$

- (ii) Time between collisions with wall $W = \frac{2l}{v}$.

- (b) (i) Remember that the magnitude of the velocity before collision is equal to that after collision but the directions are opposite.





- (iii) Momentum change per unit time for the molecule at wall W

$$\begin{aligned}
 &= \frac{\text{Momentum change}}{\text{Time between collisions with wall W}} \\
 &= \frac{2mv}{2l/v} \\
 &= \frac{mv^2}{l}
 \end{aligned}$$

- (iv) By Newton's 3rd law, average force on wall W as a result of impacts by the molecule is equal in magnitude to the momentum change per unit time for the molecule at wall W, i.e.
- $\frac{mv^2}{l}$
- .

$$\begin{aligned}
 \text{(v) Average pressure on wall W} &= \frac{\text{Average force on wall W}}{\text{Area of wall W}} \\
 &= \frac{mv^2/l}{l^2} \\
 &= \frac{mv^2}{l^3}
 \end{aligned}$$

- (c) The equation in (b)(v) is derived for the case of a single molecule moving horizontally. Suppose an ideal gas consists of
- N
- such molecules moving randomly in any three directions within the cubical container.

$$\text{For any molecule, } c^2 = c_x^2 + c_y^2 + c_z^2.$$

Since N is large and the molecules are in random motion, the mean-square-speeds,

$$\begin{aligned}
 \langle c_x^2 \rangle &= \langle c_y^2 \rangle = \langle c_z^2 \rangle = \frac{1}{3} \langle c^2 \rangle \\
 \Rightarrow \langle v^2 \rangle &= \langle c^2 \rangle = \frac{1}{3} \langle c^2 \rangle
 \end{aligned}$$

$$\therefore p = \frac{Nm}{l^3} \times \frac{1}{3} \langle c^2 \rangle = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \quad (\because V = l^3)$$

- (d) For an ideal gas,

$$\begin{aligned}
 pV &= NkT \Rightarrow p = \frac{NkT}{V} \Rightarrow \frac{NkT}{V} = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \\
 \Rightarrow kT &= \frac{1}{3} m \langle c^2 \rangle \Rightarrow T = \frac{2}{3k} \cdot \frac{1}{2} m \langle c^2 \rangle \\
 \Rightarrow T &\propto \frac{1}{2} m \langle c^2 \rangle
 \end{aligned}$$

- (e) (i) Kinetic energy as a result of the random motion of all the molecules in the gas

$$\begin{aligned}
 &= N \times \frac{1}{2} m \langle c^2 \rangle \\
 &= 3.6 \times 10^{25} \times \frac{1}{2} (4.6 \times 10^{-26}) \times 270^2 = 6.04 \times 10^4 \text{ J}
 \end{aligned}$$

- (ii) Kinetic energy of the gas as a result of being on the aircraft

$$\begin{aligned}
 &= \frac{1}{2} Mv^2 \\
 &= \frac{1}{2} (3.6 \times 10^{25} \times 4.6 \times 10^{-26}) 240^2 = 4.77 \times 10^4 \text{ J}
 \end{aligned}$$

- (iii) Internal energy of the gas = Total kinetic energies of the molecules in random motion in the gas

$$= 6.04 \times 10^4 \text{ J (from (e)(i))}$$

- (e) (iii) Note that the bulk motion of the container of gas does not affect the internal energy of the gas. In this case, potential energy from intermolecular forces between the molecules is zero.



Question 10

- (a) (i) State which **two** quantities are conserved in any collision.
 (ii) State what is meant by an *elastic collision*. [3]
- (b) An ideal gas is contained in a cylinder by means of a piston, as shown in Fig. 3.1.

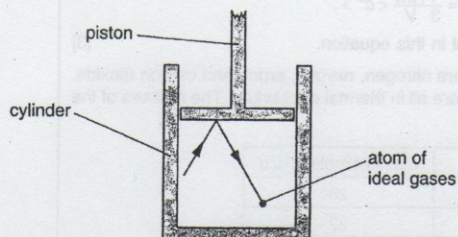


Fig. 3.1

The cylinder and piston are made of a thermal insulator. An atom of the gas collides with the piston, as illustrated.

State, with a reason, whether the momentum of the atom is conserved in the collision. [2]

- (c) The piston in (b) is lowered so that the volume of the gas is reduced.
- (i) State what change, if any, occurs in the speed of a gas atom during an elastic collision with the moving piston. [1]
- (ii) Use the kinetic theory of gases to explain why the temperature of the gas rises. [2]
- (iii) Write down a word equation to represent the conservation of energy for this change in volume of the gas. [2]

[D03/P2/Q3]

Suggested Solution:

- (a) (i) 1. momentum
 2. total energy
- (ii) An *elastic collision* is a collision in which the total kinetic energy of the colliding bodies is the same after the collision as it was before.
- (b) The momentum of the atom is conserved in the collision because the piston is stationary.
- (c) (i) The speed of a gas atom increases after it collides elastically with the moving piston.
- (ii) According to the kinetic theory of gases, the temperature of a gas is proportional to the average kinetic energy of the atoms of the gas. Since the speeds of the gas atoms increase after colliding with the moving piston, the average kinetic energy of the gas atoms increase and the temperature of the gas hence increases.
- (iii) Increase in internal energy = Pressure \times Decrease in volume of the gas
- (c) (iii) The first law of thermodynamics,

$$\Delta U = \Delta Q + \Delta W$$
 where ΔU is the increase in internal energy.
 ΔQ is the heat supplied to system.
 ΔW is the work done on system.

**Question 11**

- (a) State **three** basic assumptions of the kinetic theory of gases. [3]
 (b) It can be shown from the kinetic theory that the pressure p exerted by a gas is given by the relationship.

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

State the meaning of each symbol in this equation. [3]

- (c) The four main constituents of air are nitrogen, oxygen, argon and carbon dioxide. The gases, assumed to be ideal, are all in thermal equilibrium. The masses of the molecules are listed in Fig. 3.1.

	mass of molecule/ u
nitrogen	28
oxygen	32
argon	40
carbon dioxide	44

Fig. 3.1

- (i) State what is meant by thermal equilibrium. [1]
 (ii) Show that the mean-square speed of the molecules in air is inversely proportional to the mass of each molecule. [3]
 (iii) State which gas has the molecules with the smallest root-mean-square speed. [1]
 (iv) The root-mean-square speed of oxygen molecules is 480 ms^{-1} . Calculate the root-mean-square speed of nitrogen molecules. [3]
 (d) Describe the movement of a particle of dust undergoing Brownian motion in air. [2]
 (e) Estimate the root-mean-square speed of a particle of dust, of mass $1.2 \times 10^{-12} \text{ kg}$, in the air. State any assumption you make. [4]

[D03/P3/Q3]

Suggested Solution:

- (a) Three basic assumptions of the kinetic theory of gases:
 1. The attraction between the molecules is negligible.
 2. The molecules behave as perfectly elastic spheres.
 3. The duration of a collision is negligible compared with the time between collisions.
 (b) N : number of molecules in the gas.
 m : mass of each molecule.
 V : volume occupied by the gas.
 $\langle c^2 \rangle$: mean-square speed of the gas molecules.
 (c) (i) Two objects are said to be in thermal equilibrium when heat does not flow from one object to another when they are placed together. This is the situation when the two objects are at the same temperature.

$$(ii) p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \Rightarrow pV = \frac{1}{3} Nm \langle c^2 \rangle$$

$$\text{The gases are ideal gases} \Rightarrow pV = NkT$$

$$\Rightarrow \frac{1}{3} Nm \langle c^2 \rangle = NkT$$

$$\Rightarrow \frac{1}{3} m \langle c^2 \rangle = kT$$

$$\Rightarrow \langle c^2 \rangle = \frac{3kT}{m}$$

$$\Rightarrow \langle c^2 \rangle \propto \frac{1}{m}$$

where k is a constant.

T is constant since the gases are in thermal equilibrium.





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(iii) Carbon dioxide

$$(iv) \langle c^2 \rangle \propto \frac{1}{m} \Rightarrow \sqrt{\langle c^2 \rangle} \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{\sqrt{\langle c_O^2 \rangle}}{\sqrt{\langle c_N^2 \rangle}} = \frac{\sqrt{m_N}}{\sqrt{m_O}}$$

$$\Rightarrow \sqrt{\langle c_N^2 \rangle} = \sqrt{\frac{m_O}{m_N}} \times \sqrt{\langle c_O^2 \rangle} = \sqrt{\frac{32}{28}} \times 480 = 513 \text{ ms}^{-1}$$

(d) A particle of dust undergoing Brownian Motion in air will undergo constant, random motion due to continual bombardment with air molecules.

(e) From part (c)(ii),

$$\langle c^2 \rangle = \frac{3kT}{m}$$

$$\Rightarrow \sqrt{\langle c^2 \rangle} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times (273 + 30)}{1.2 \times 10^{-12}}} = 1.02 \times 10^{-4} \text{ ms}^{-1}$$

Assumption : The temperature of the air is 30 °C.

(c) (iii) Of the four molecules, carbon dioxide has the largest mass. Hence it has the smallest root-mean-square speed since $\langle c^2 \rangle \propto \frac{1}{m}$.

(iv) Since $\langle c^2 \rangle \propto \frac{1}{m}$
 $\sqrt{\langle c^2 \rangle} \propto \frac{1}{\sqrt{m}}$

∴ the molecule with the largest mass has the smallest root-mean-square speed.

Question 12

(a) Put the following terms into pairs, so that in each pair a quantity is given followed by its corresponding unit.

- coulomb
- power
- volt
- magnetic flux density
- work
- watt
- potential difference
- charge
- tesla
- joule

[4]

(b) Explain why it is technically incorrect to define speed as distance travelled per second. Include in your answer the correct statement defining speed. [2]

(c) Suggest why, in the SI system, it was **not** chosen to define potential difference as the product of electric current and resistance. [2]

(d) (i) By putting each quantity into base units, show that the equation

$$pV = nRT$$

is homogeneous. [4]

(ii) Explain why a homogeneous equation may not necessarily be correct. [1]

(e) The pressure of an ideal gas, as derived from the kinetic theory, is given by the equation

$$p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle.$$

(i) State the meaning of each of the symbols N , m and $\langle c^2 \rangle$. [2]

(ii) Use this equation, together with the one in (d)(i), to calculate the root mean square speed of the oxygen molecules in air at a temperature of 27 °C. The mass of one mole of oxygen molecules is 32 g. [3]

(iii) Assuming the gas to behave ideally, calculate the internal energy of a mole of oxygen at this temperature. [2]

[D04/P3/Q1]





Suggested Solution:

Quantity	Unit
charge	coulomb
magnetic flux density	tesla
potential difference	volt
power	watt
work	joule

- (b) Physical quantities should be defined in terms of other physical quantities. 'Second' is not a physical quantity but is a unit. Hence, speed should be defined as distance travelled per unit time.
- (c) The reason for not defining potential difference this way could be due to the fact that such a definition cannot be generalised to a situation where charges are moved from one point to another in a vacuum instead of in a resistor or any material for which resistance is defined.

(d) (i) Base units of left-hand-side of equation = (Base units of p)(Base units of V)

$$= \frac{\text{kg ms}^{-2}}{\text{m}^2} \times \text{m}^3$$

$$= \text{kg m}^2 \text{s}^{-2}$$

Base units of right-hand-side of equation = (Base units of n) \times (Base units of R) \times (Base units of T)

$$= \text{mol} \times \frac{\text{kg ms}^{-2} \times \text{m}}{\text{mol} \times \text{K}} \times \text{K}$$

$$= \text{kg m}^2 \text{s}^{-2}$$

Since base units of both sides of the equation are the same, the equation is homogeneous.

- (ii) A homogeneous equation may not necessarily be correct because certain quantities such as refractive index etc. do not have base units. The rightful presence or absence of such quantities hence cannot be ascertained through checking for homogeneity of the equation.
- (e) (i) N : number of molecules in the ideal gas
 m : mass of each molecule
 $\langle c^2 \rangle$: mean-square-speed of the molecules

$$(ii) p = \frac{1}{3} \frac{Nm}{V} \langle c^2 \rangle \Rightarrow pV = \frac{1}{3} Nm \langle c^2 \rangle$$

$$\Rightarrow nRT = \frac{1}{3} Nm \langle c^2 \rangle \quad \text{since } pV = nRT.$$

$$\Rightarrow \frac{N}{N_A} RT = \frac{1}{3} Nm \langle c^2 \rangle \quad \text{where } N_A : \text{Avogadro's number.}$$

$$\Rightarrow \langle c^2 \rangle = \frac{3RT}{N_A m}$$

$$\Rightarrow \sqrt{\langle c^2 \rangle} = \sqrt{\frac{3RT}{N_A m}} \quad \text{where } N_A m = \text{mass of one mole of the gas.}$$

$$= \sqrt{\frac{3 \times 8.31 \times (273.16 + 27)}{32 \times 10^{-3}}} = 484 \text{ ms}^{-1}$$

- (iii) For an ideal gas, internal energy = K.E. of each molecule \times number of molecules
 \Rightarrow For one mole of oxygen at this temperature,

$$\text{internal energy} = \frac{1}{2} m \langle c^2 \rangle \times N_A$$

$$= \frac{1}{2} \times N_A m \times \langle c^2 \rangle = \frac{1}{2} \times 32 \times 10^{-3} \times 484^2 = 3.74 \times 10^3 \text{ J}$$

(d) (i)

$$\text{Base units of } p = \frac{\text{Base units of force}}{\text{Base units of area}}$$

$$(\therefore \text{Pressure} = \frac{\text{Force}}{\text{Area}})$$

From table of constants,

$$R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$$

 \Rightarrow Base units of R

$$= \frac{\text{Base units of work}}{\text{Base units of amount of substance} \times \text{Base units of temperature}}$$



**Question 13**

- (a) The ideal gas equation is $pV = nRT$. Explain why non-SI units may be used for p and V but the temperature T cannot have the unit $^{\circ}\text{C}$. [2]
- (b) Write down the exact temperature on the Kelvin scale of zero degrees Celsius. [1]
- (c) In an attempt to beat the world altitude record for a balloon, a helium balloon containing $15\,000\text{ m}^3$ of helium at a temperature of 288 K was launched from sea level, where the pressure of the gas was 101 kPa . The balloon, carrying a payload, rose to an altitude of 32.0 km before reaching equilibrium. Data concerning atmospheric conditions are given in the table.

	sea level altitude = 0	equilibrium altitude = 32.0 km
pressure of helium	101 kPa	0.890 kPa
temperature	288 K	228 K
density of air	1.23 kg m^{-3}	0.0134 kg m^{-3}

Calculate

- (i) the volume of helium at 32.0 km , [2]
- (ii) the weight of air displaced by the balloon at equilibrium altitude, [2]
- (iii) the total weight of balloon, helium and payload, [1]
- (iv) the resultant force on the balloon at sea level, [2]
- (v) the acceleration of the balloon at take-off. [2]
- (d) (i) Using the equations $pV = \frac{1}{3}Nm\langle c^2 \rangle$ and $pV = NkT$, derive an expression for the relationship between the average translational kinetic energy of a helium atom and the temperature. [1]
- (ii) Hence find the average translational kinetic energy of one of the helium atoms in the balloon in (c), when the balloon is at an altitude of 32.0 km . [1]
- (iii) Calculate the amount, in mol, of helium in the balloon. [2]
- (iv) Assuming that the gas behaves as an ideal gas, calculate the kinetic energy of all the helium at equilibrium altitude. [2]
- (e) Suggest why the change in the potential energy of the gas in the balloon as it rises does not change its internal energy. [2]

[N05/P3/Q2]

Suggested Solution:

- (a) For temperature at 0°C , this would mean $pV = 0$ which cannot be true as there are pressure and volume present in an ideal gas.
- (b) 273.15 K
- (c) (i) Let V be volume of helium at 32.0 km .

$$\frac{pV}{T} = \text{constant}$$

$$\frac{(101\text{ kPa})(1.5 \times 10^4)}{288} = \frac{(0.89\text{ kPa})V}{228}$$

$$V = 1.35 \times 10^6\text{ m}^3$$

- (ii) Weight of air displaced = mg
- $$= \rho \cdot V \cdot g$$
- $$= (0.0134)(1.35 \times 10^6)(9.81) = 1.77 \times 10^5\text{ N}$$





- (iii) Total weight of balloon, helium and payload = Weight of air displaced
 $= 1.77 \times 10^5 \text{ N}$
- (iv) Resultant force = Upthrust – Weight
 $F = (\text{Weight of displaced air at sea level}) - \text{Weight}$
 $= \rho \cdot V \cdot g - 1.77 \times 10^5$
 $= (1.23)(15000)(9.81) - 1.77 \times 10^5$
 $= 3.99 \times 10^3 \text{ N}$
- (v) $F = ma$
 $\Rightarrow a = \frac{F}{m} = \frac{3.99 \times 10^3 \times 9.81}{1.77 \times 10^5} = 0.221 \text{ ms}^{-2}$
- (d) (i) $\frac{1}{3}Nm < c^2 > = NkT$
 $\frac{1}{2}m < c^2 > = \frac{3}{2}kT$
- (ii) $K.E. = \frac{3}{2}kT$
 $= \frac{3}{2}(1.38 \times 10^{-23})(228) = 4.72 \times 10^{-21} \text{ J}$
- (iii) Number of moles, $n = \frac{\text{Mass}}{\text{Molar mass}}$
 $= \frac{\rho V}{(4 \times 10^{-3})} = \frac{(1.23)(15000)}{4 \times 10^{-3}}$
 $= 4.61 \times 10^6 \text{ moles}$
- (iv) Total K.E. of helium = (Total no. of helium) \times (K.E. of 1 He atom)
 $= (n \cdot N_A) \times K.E.$
 $= (4.61 \times 10^6)(6.02 \times 10^{23})(4.72 \times 10^{-21})$
 $= 1.31 \times 10^{10} \text{ J}$
- (e) The increase in height changes the gravitational potential energy of the balloon and not the intermolecular potential energy which affects internal energy of gas.

Question 14

- (a) An amount of 1.00 mol of Helium-4 gas is contained in a cylinder at a pressure of $1.02 \times 10^5 \text{ Pa}$ and a temperature of 27°C .
- (i) Calculate the volume of gas in the cylinder. [2]
- (ii) Hence show that the average separation of gas atoms in the cylinder is approximately $3.4 \times 10^{-9} \text{ m}$. [2]
- (b) Calculate
- (i) the gravitational force between two Helium-4 atoms that are separated by a distance of $3.4 \times 10^{-9} \text{ m}$, [3]
- (ii) the ratio

$$\frac{\text{weight of a Helium-4 atom}}{\text{gravitational force between two Helium-4 atoms with separation } 3.4 \times 10^{-9} \text{ m}}$$
 [2]
- (c) Comment on your answer to (b)(ii) with reference to one of the assumptions of the kinetic theory of gases. [2]

[N07/P4/Q2]



**Suggested Solution:**

(a) (i) $PV = nRT$

$$(1.02 \times 10^5)V = (1.00)(8.31)(27 + 273.15)$$

$$V = 0.0244$$

$$\therefore \text{volume} = 0.0244 \text{ m}^3$$

(ii) Volume of 1 mole of gas = 0.0244 m^3

or Volume of 6.02×10^{23} atoms of gas = 0.0244 m^3

Volume of 1 atom of gas = $\frac{0.0244}{6.02 \times 10^{23}} = 4.05 \times 10^{-26} \text{ m}^3$

$$\therefore \text{Separation} = (4.05 \times 10^{-26})^{\frac{1}{3}} = 3.43 \times 10^{-9} \text{ m}$$

(b) (i) $F = G \frac{Mm}{r^2}$

$$= \frac{(6.67 \times 10^{-11})(4 \times 1.66 \times 10^{-27})(4 \times 1.66 \times 10^{-27})}{(3.4 \times 10^{-9})^2} = 2.54 \times 10^{-46}$$

$$\therefore \text{force} = 2.54 \times 10^{-46} \text{ N}$$

(ii) $\frac{\text{weight of a Helium-4 atom}}{\text{gravitational force between two Helium-4 atoms with separation } 3.4 \times 10^{-9} \text{ m}}$

$$= \frac{mg}{F_G}$$

$$= \frac{(4 \times 1.66 \times 10^{-27})(9.81)}{2.54 \times 10^{-46}}$$

$$= 2.56 \times 10^{20}$$

$$\therefore \text{ratio} = 2.6 \times 10^{20}$$

- (c) The assumption is that forces between atoms are negligible. So the gravitational force is very small as compared to weight.

Question 15

- (a) Explain qualitatively how molecular movement causes the pressure exerted by a gas. [3]

- (b) The density of neon gas at a temperature of 273 K and a pressure of
- 1.02×10^5
- Pa is
- 0.900 kg m^{-3}
- . Neon may be assumed to be an ideal gas.

Calculate the root-mean-square (r.m.s.) speed of neon atoms at

(i) 273 K, [3]

(ii) 546 K. [2]

- (c) The calculations in (b) are based on the density for neon being
- 0.900 kg m^{-3}
- .

Suggest the effect, if any, on the root-mean-square speed of changing the density at constant temperature. [2]

[J08/P4/Q2]

- (a) (i) Temperature must be changed into kelvin scale
-
- $PV = nRT$
- .

- (ii) The average separation between two atoms is equal to its size. If
- d
- is the separation then the volume of the molecule (assuming spherical) is given by

$$V = \frac{4}{3}\pi\left(\frac{d}{2}\right)^3$$

$$V \propto d^3$$

$$d \propto (V)^{\frac{1}{3}}$$



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Suggested Solution:

(a) The molecules of a gas move randomly and rebound back after collisions with the walls of container. Therefore change of momentum per unit time of large number of molecules define force acting per unit area.

(b) (i) $p = \frac{1}{3} \rho < c^2 >$
 $1.02 \times 10^5 = \frac{1}{3} (0.900) < c^2 >$
 $< c^2 > = \frac{3(1.02 \times 10^5)}{0.900} = 3.40 \times 10^5$
 $c_{r.m.s} = \sqrt{< c^2 >} = \sqrt{3.40 \times 10^5} = 583$
 $\therefore \text{speed} = 583 \text{ ms}^{-1}$

(ii) Since $< c^2 > \propto T$
 $\frac{< c_2^2 >}{< c_1^2 >} = \frac{T_2}{T_1}$
 $\frac{< c_2^2 >}{3.40 \times 10^5} = \frac{546}{273}$
 $< c_2^2 > = 6.80 \times 10^5$
 $c_{r.m.s} = \sqrt{< c_2^2 >} = \sqrt{6.80 \times 10^5} = 824.6$
 $\therefore \text{speed} = 825 \text{ ms}^{-1}$

(c) Since root-mean-square speed depends only on temperature so there is no effect due to constant temperature.

(a) Pressure results from the averaging of the forces due to a very large number of collisions per unit time with the walls of container.

(b) (i) Pressure of an Ideal gas

$$p = \frac{1}{3} \frac{Nm}{V} < c^2 >$$

$$\text{As } \frac{Nm}{V} = \rho$$

$$\text{So } p = \frac{\rho}{3} < c^2 >$$

(ii) Average $E_K \propto T$ by relation

$$\frac{1}{2} m < c^2 > = \frac{3}{2} nkT$$

Question 16

Some smoke particles are viewed through a microscope, as illustrated in Fig. 5.1.

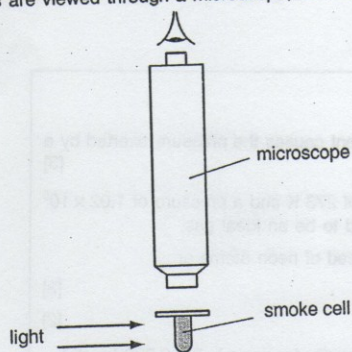


Fig. 5.1

Brownian motion is observed.

- (a) Explain what is meant by *Brownian motion*. [2]
- (b) Suggest and explain why Brownian motion provides evidence for the movement of molecules as assumed in the kinetic theory of gases. [2]
- (c) Smoke from a poorly maintained engine contains large particles of soot. Suggest why the Brownian motion of such large particles is undetectable. [2]

[N08/P2/Q5]



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Suggested Solution:

- (a) The perpetual random motion of particles of smoke is called *Brownian motion*.
- (b) Abrupt change of path of smoke particles is due to uneven bombardment of invisible air particles from different directions and at different times. This indicates the random motion of molecules of gases.
- (c) Particles of soot are massive and need a greater force to change their path or to accelerate them. The less force due to collision causes only small movement which is difficult to detect.

- (a) Do not write movement of molecules or atoms. Particles is more appropriate word.
- (b) Random motion means irregular motion in any direction (space) and at any time.

Question 17

Two deuterium (${}^2_1\text{H}$) nuclei are travelling directly towards one another. When their separation is large compared with their diameters, they each have speed v as illustrated in Fig. 5.1.

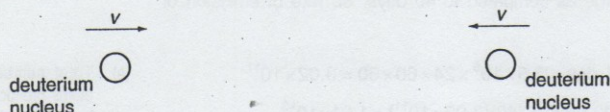


Fig. 5.1

The diameter of a deuterium nucleus is 1.1×10^{-14} m.

- (a) Use energy considerations to show that the initial speed v of the deuterium nuclei must be approximately 2.5×10^6 m s $^{-1}$ in order that they may come into contact. Explain your working. [3]
- (b) For a fusion reaction to occur, the deuterium nuclei must come into contact. Assuming that deuterium behaves as an ideal gas, deduce a value for the temperature of the deuterium such that the nuclei have an r.m.s. speed equal to the speed calculated in (a). [4]
- (c) Comment on your answer to (b). [1]

[N08/P4/Q5]

Suggested Solution:

- (a) Loss of Kinetic energy of both nuclei = Gain in Electric Potential energy

$$2 \left[\frac{1}{2} m v^2 \right] = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{r}$$

$$2 \left[\frac{1}{2} (2m_p) v^2 \right] = \frac{1}{4\pi\epsilon_0} \frac{Q^2}{r}$$

$$2 \left[\frac{1}{2} (2)(1.67 \times 10^{-27}) \right] v^2 = \frac{1}{4(3.14)(8.85 \times 10^{-12})} \left[\frac{(1.60 \times 10^{-19})^2}{1.1 \times 10^{-14}} \right]$$

$$v^2 = 6.27 \times 10^{12} \quad \Rightarrow \quad v = 2.50 \times 10^6 \text{ ms}^{-1}$$

- (b) Since $pV = \frac{1}{3} Nm \langle c^2 \rangle$ and $pV = NKT$

By comparison

$$\frac{1}{3} Nm \langle c^2 \rangle = NKT$$

$$\frac{1}{3} m \langle c_{r.m.s} \rangle^2 = KT$$

$$\frac{1}{3} [2(1.67 \times 10^{-27})] (2.50 \times 10^6)^2 = (1.38 \times 10^{-23}) T \quad \Rightarrow \quad T = 5.04 \times 10^8 \text{ K}$$

- (c) This is a very high temperature that causes Fusion reaction for star formation.

- (a) Deuterium ${}^2_1\text{H}$ has two protons in its nucleus $m = 2m_p$

- (b) $c_{r.m.s} = \sqrt{\langle c^2 \rangle}$ as $\langle c^2 \rangle$ is the mean value of the squared speed of particles of an ideal gas.





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Question 18Sources of α -particles are frequently found to contain traces of helium gas.A radioactive source emits α -particles at a constant rate of $3.5 \times 10^6 \text{ s}^{-1}$. The α -particles are collected for a period of 40 days. Each α -particle becomes one helium atom.

- (a) By reference to the half-life of the source, suggest why it may be assumed that the rate of emission of α -particles is constant. [1]
- (b) The helium gas may be assumed to be an ideal gas. Calculate the volume of gas that is collected at a pressure of $1.5 \times 10^5 \text{ Pa}$ and at a temperature of 17°C . [3]

[J09/P4/Q2]

Suggested Solution:

- (a) Half life of source is very large as compared to 40 days. So rate of emission of α -particles is constant.

(b) Number of Helium atom in 1 day = $3.5 \times 10^6 \times 24 \times 60 \times 60 = 3.02 \times 10^{11}$

Number of Helium atom in 40 days = $(40)(3.02 \times 10^{11}) = 1.21 \times 10^{13}$

$$PV = NKT$$

$$(1.5 \times 10^5)V = (1.21 \times 10^{13})(1.38 \times 10^{-23})(17 + 273.15)$$

$$V = 3.23 \times 10^{-13} \text{ m}^3$$

- (b) Total number of Helium atoms produced in 40 days is considered.

