You should try to answer on your own before seeking *H.E.L.P.S.*

TOPIC opening ....

Question 1

An aircraft flies with its wings tilted as shown in Fig. 3.1 in order to fly in a horizontal circle of radius *r*. The aircraft has mass 4.00×10^4 kg and has a constant speed of 250 ms^{-1}.

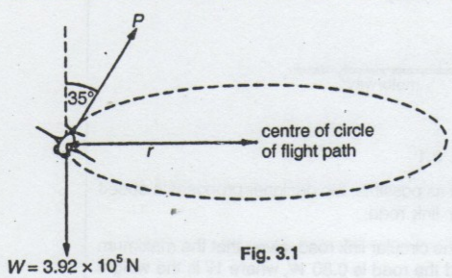


Fig. 3.1

With the aircraft flying in this way, two forces acting on the aircraft in the vertical plane are the force *P* acting at an angle of 35° to the vertical and the weight *W*.

(a) State the vertical component of *P* for horizontal flight.

vertical component of *P* = ... N [1]

(b) Calculate *P*.

P = ... N [2]

(c) Calculate the horizontal component of *P*.

horizontal component of *P* = ... N [1]

(d) Use Newton's second law to determine the acceleration of the aircraft towards the centre of the circle.

acceleration = ... ms^{-2} [2]

(e) Calculate the radius *r* of the path of the aircraft's flight.

r = ... m [2]

[J00/P2/Q3]

Suggested Solution:

(a) vertical component of $P = 3.92 \times 10^5$ N

(b) $P\cos 35° = 3.92 \times 10^5$

$$P = \frac{3.92 \times 10^5}{\cos 35°} = 4.79 \times 10^5 \text{ N}$$

(c) horizontal component of $P = P\sin 35°$

$$= 4.79 \times 10^5 \sin 35° = 2.74 \times 10^5 \text{ N}$$

(d) By Newton's second law,

horizontal component of $P = ma$

$$a = \frac{2.74 \times 10^5}{4.00 \times 10^4} = 6.86 \text{ ms}^{-2}$$

(e) $a = \frac{v^2}{r}$

$$r = \frac{v^2}{a} = \frac{(250)^2}{6.86} = 9.1 \times 10^3 \text{ m}$$

(a) Since the aircraft is in horizontal flight, it is in equilibrium in the vertical direction. Hence the vertical component of *P* should equal the weight of the aircraft.

(e) In circular motion of object, though the tangential speed may be constant, there is a net acceleration towards centre of the circular path given by

$$a = \frac{v^2}{r} = r\omega^2$$

where v = speed of object.
r = radius of the circular path.
$\omega = \frac{v}{r}$ = angular speed of the object.

Question 2

(a) An object travelling at a constant speed in a circular path is said to have a centripetal acceleration. Explain, using a diagram,

(i) why there is an acceleration even though the speed is constant.

(ii) the direction of the acceleration. [4]

(b) A motorway designer plans to have motorists leaving one motorway and joining another by constructing a circular link road, as shown in Fig. 3.1.

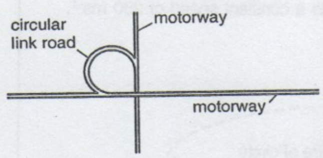


Fig. 3.1

In order to use as small an area of land as possible, the designer proposes a speed limit of 25 m^{-1} for cars on the circular link road.

(i) Calculate the minimum radius for the circular link road, given that the maximum sideways force between a car and the road is 0.80 *W*, where *W* is the weight of a car. [3]

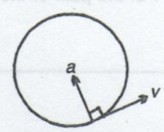
(ii) Suggest why lorries may have to go at a slower speed than the 25 ms^{-1} limit for cars. [2]

[J01/P2/Q3]

Suggested Solution:

(a) (i) *Newton's First Law of Motion* states that an object will remain in its state of rest or of uniform motion in a straight line unless an external force acts on it. Hence, an object travelling in a circular path must be acted upon by a force which changes the direction of motion of the object continuously. Since the object experiences a net force, there is an acceleration, according to Newton's Second Law of Motion.

(ii) The object moves with constant speed, implying that the net force acting on the object must be perpendicular to its path so that the force changes only the direction of the motion but not its tangential speed. Hence, the direction of the acceleration is perpendicular to the tangential velocity i.e. the acceleration is directed to the centre of the circular path of the object.



(b) (i) $F = \frac{mv^2}{r}$

$$r = \frac{mv^2}{F} = \frac{mv^2}{0.80W} = \frac{mv^2}{0.80mg} = \frac{v^2}{0.80g} = \frac{25^2}{0.80 \times 9.81} = 79.6 \text{ m}$$

(ii) The centripetal force required for travelling in a circular path is $F = \frac{mv^2}{r}$.

A lorry has a larger mass, *m*, than a car. Hence, in order for a lorry to travel in a circular path of the same radius, the lorry would have to travel at a slower speed than the 25 ms^{-1} limit for cars, assuming the same centripetal force.

(b) (i) The sideways force between the car and the road provides the centripetal force required for the car to travel in a circular path. Hence,

$F = \frac{mv^2}{r}$

where $F = 0.80\ W$.

In other words, if the lorry enters the circular link road at the same speed as that of a car, it will require a radius that is larger than that required for a car. If the turning radius is larger than 79.6 m, the lorry may veer off the road and cause an accident.

Question 3

(a) An object rotates in a vertical circle. Which of the following quantities are constant when the object has constant speed?

PERIOD, FREQUENCY, ANGULAR VELOCITY, VELOCITY, ACCELERATION, KINETIC ENERGY, POTENTIAL ENERGY. [4]

(b) Fig. 2.1 shows a ride in an amusement park.

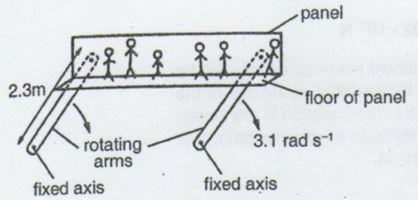
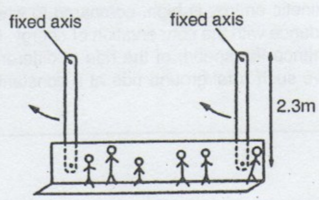


Fig. 2.1(a)



(Fig. 2.1(b)

The passengers are strapped firmly to a vertical panel. The floor of the panel stays horizontal while it is being moved by two arms rotating with angular velocity 3.1 rad s^{-1}. Each passenger rotates in a vertical circle of radius 2.3 m. Calculate

(i) the time taken for one revolution of the arms,

(ii) the linear speed of each passenger,

(iii) the centripetal acceleration of each passenger. [5]

c) Consider a person of mass 60 kg on the ride described in **(b)** at the instant when the panel is at the bottom of the path, as shown in Fig. 2.1(b).

(i) State the direction of the acceleration of this person.

(ii) Calculate the resultant force necessary to cause this acceleration.

(iii) Draw a force diagram showing the weight *W* of the person and the force *P* which the panel exerts on the person.

(iv) Calculate *P*. [7]

(d) Using energy considerations, suggest why it is difficult to drive a fairground ride such as described in **(b)** at a constant speed. [4]

[D02/P3/Q2]

Suggested Solution:

(a) Period, Frequency, Angular velocity, Kinetic energy will be constant when the object has constant speed.

(b) (i) $\omega = \frac{2\pi}{T}$

$T = \frac{2\pi}{\omega} = \frac{2\omega}{3.1} = 2.03$ s

(ii) Linear speed of each passenger, $v = \omega r$

$= 3.1 \times 2.3 = 7.13$ ms^{-1}

(iii) Centripetal acceleration of each passenger, $a = \frac{v^2}{r}$

$= \frac{7.13^2}{2.3} = 22.1$ ms^{-2}

(c) (i) Vertical upwards

(ii) F = mass of person × centripetal acceleration

$= 60 \times 22.1 = 1.33 \times 10^3$ N

(a) Velocity is not constant because the direction of the velocity of the object is tangential to the circular path and hence changes.
The acceleration is always directed toward the centre of the circular path.
While the magnitudes of the velocity and acceleration do not change, their directions do.
Potential energy changes as the height of the object above the ground changes as it rotates.

(b) (i) Note that the angular velocity, period, linear velocity and centripetal acceleration of the panel are the same as that for the passengers.

(c) The centripetal acceleration is always directed towards the centre of the circular path.

(iii)

(iv) Since the resultant force on the person is directed vertically upward,

$F = P - W$

$P = F + W = 1.33 \times 10^3 + (60 \times 9.81) = 1.92 \times 10^3$ N

(d) When the panel is at the bottom of the path, the gravitational potential energy will be low and the kinetic energy is high, compared to when the panel is at the top of the path, in accordance with the conservation of energy. Due to the variation in the kinetic energy, and hence the speed, of the ride at different points in its circular path, it is difficult to drive such a fairground ride at a constant speed.

Question 4

An α-particle and a β-particle are both travelling along the same path at a speed of 1.5×10^6 ms^{-1}.

They then enter a region of uniform magnetic field as shown in Fig. 5.1.

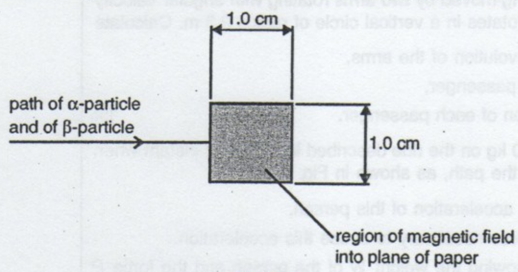


Fig. 5.1

The magnetic field is normal to the path of the particles and is into the plane of the paper.

(a) Show that, for a particle of mass m and charge q travelling at speed v normal to a magnetic field of flux density B, the radius r of its path in the field is given by

$$r = \frac{mv}{Bq}.$$ [3]

(b) Calculate the ratio

$$\frac{\text{radius of path of the } \alpha\text{-particle}}{\text{radius of path of the } \beta\text{-particle}}$$ [3]

(c) The magnetic field has flux density 1.2mT. Calculate the radius of the path of

(i) the α-particle,

(ii) the β-particle. [3]

(d) The magnetic field extends over a region having a square cross-section of side 1.0cm (see Fig. 5.1). Both particles emerge from the region of the field.

On Fig. 5.1,

(i) mark with the letter **A** the position where the emergent α-particle may be detected,

(ii) mark with the letter **B** the position where the emergent β-particle may be detected. [3]

[J03/P4/Q5]

Suggested Solution:

(a) A charged particle follows a circular path when travelling in a magnetic field under such circumstances.

Centripetal force required = Magnetic force

$$\frac{mv^2}{r} = Bqv$$

$$r = \frac{mv^2}{Bqv} = \frac{mv}{Bq}$$

(b) For the α-particle, $r_\alpha = \frac{m_\alpha v}{Bq_\alpha}$

For the β-particle, $r_\beta = \frac{m_\beta v}{Bq_\beta}$

$$\Rightarrow \frac{r_\alpha}{r_\beta} = \frac{m_\alpha q_\beta}{m_\beta q_\alpha} = \frac{(2m_p+2m_n)e}{m_e.2e}$$

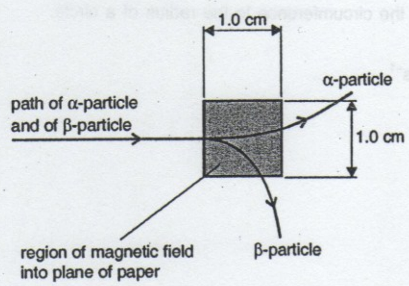
$$= \frac{(4 \times 1.66 \times 10^{-27}) \times 1}{9.11 \times 10^{-31} \times 2} = 3.64 \times 10^3$$

(b) An α-particle is a Helium nucleus, made up of 2 protons and 2 neutrons. It has a charge of +2e (assume $m_n = m_p$).
A β-particle is an electron. It has a charge of −1e.

(c) (i) $r_\alpha = \frac{m_\alpha v}{Bq_\alpha} = \frac{(4 \times 1.66 \times 10^{-27}) \times 1.5 \times 10^6}{(1.2 \times 10^{-3}) \times 2 \times 1.60 \times 10^{-19}} = 25.9$ m

(ii) $r_\beta = \frac{m_\beta v}{Bq_\beta} = \frac{(9.11 \times 10^{-31}) \times 1.5 \times 10^6}{(1.2 \times 10^{-3}) \times 1.60 \times 10^{-19}} = 7.12 \times 10^{-3}$ m

(d)



Question 5

(a) Define the *radian*. [2]

(b) The path of an α-particle travelling at a linear speed of 2.8 × 10⁷ ms⁻¹ is an arc of a circle of radius 1.4m, as shown in Fig. 2.1.

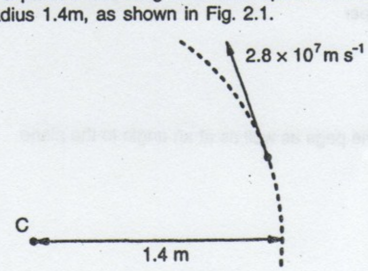


Fig. 2.1

(i) Calculate the angular speed of the α-particle about the centre C of the circle.

angular speed = ... rad s⁻¹ [2]

(ii) The α-particle is moving in a uniform force-field.

1. State the name of the force-field that would produce this motion. [1]
2. State the direction of the force-field. [1]
3. Calculate the magnitude of the field strength.

field strength = ... [4]

(c) The track of a second α-particle in the same force-field is seen to spiral, as shown in Fig. 2.2.

Fig. 2.2

Suggest a reason for the spiral path and hence, on Fig. 2.2, mark the direction of motion of the α-particle. [3]

[N06/P2/Q2]

Suggested Solution:

(a) The radian is the unit for the ratio of the circumference to the radius of a circle.

(b) (i) $\omega = \frac{v}{r} = \frac{2.8\times10^{7}}{1.4} = 2.0\times10^{7}\ \text{rad s}^{-1}$

(ii) 1. Magnetic

2. Into the plane of paper

3. mass of $\alpha = 4u$

charge of $\alpha = 2e$

$$Bqv = \frac{mv^2}{r}$$

$$\Rightarrow\ B = \frac{mv}{qr} = \frac{m}{q}(\omega) = \frac{4\times1.66\times10^{-27}}{2\times1.6\times10^{-19}}(2.0\times10^{7})$$

$$= 0.42\ \text{T}$$

(c)

The motion of particle is at an angle into the page as well as at an angle to the plane of the magnetic field.

Question 6

(a) An object travelling in a circle of radius r at constant speed v is accelerating. By drawing a vector diagram to show the combination of vectors, explain how this is possible. [2]

(b) An object P moves at a constant speed v through an arc of a circle of radius r. The arc subtends an angle 0.010 rad at the centre of the circle, as shown in Fig.3.1.

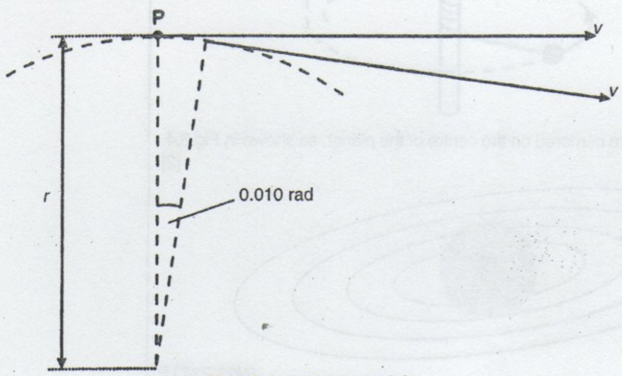


Fig. 3.1. (not to scale)

(i) Determine, in terms of v, the magnitude of the change in velocity. [1]

(ii) State the direction of the acceleration of P. [1]

(iii) Deduce, in terms of r and v, the time taken for P to travel 0.010 rad. [2]

(iv) Hence show that the magnitude of the acceleration of P is $\frac{v^2}{r}$. [1]

(c) A theme park ride is illustrated in Fig. 3.2. The carriages accelerates down the slope and then loop the loop on a circular section of track. The radius of the circular section of track is 8.6 m.

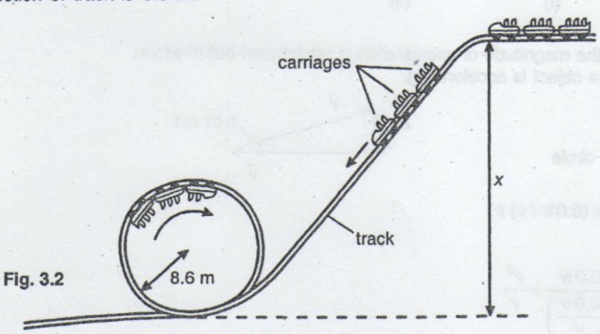


Fig. 3.2

(i) Find the minimum speed of the carriages at the top of the circular track so that the carriages remain in contact with the track. [2]

(ii) In practice, it is essential for designers to build in a considerable safety margin. Each carriage and its passengers has total mass 800 kg. At the top of the loop, the carriage travels at 17 ms^{-1}. Calculate the force that the track exerts on the carriage at the top of the loop. [3]

(d) For the carriage in **(c)** to have a speed of 17 ms^{-1} at the top of the loop, it must have fallen from a height of at least x.

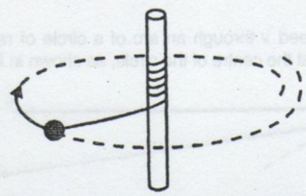
(i) Deduce a value for x. [3]

(ii) State an assumption that you made in making this deduction. [1]

(e) Forces acting on bodies which travel in a circle are responsible for the following. Suggest an explanation for each.

(i) A string may snap if it is attached to an object and the object is spun around a vertical pole as shown in Fig. 3.3. [2]

Fig. 3.3



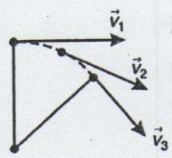
(ii) The rings of Saturn are centered on the centre of the planet, as shown in Fig.3.4. [2]

Fig. 3.4

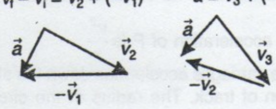
[N06/P3/Q3]

Suggested Solution:

(a)



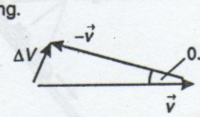
$\vec{a} = \vec{v}_1 - \vec{v}_1 = \vec{v}_2 + (-\vec{v}_1)$ $\vec{a} = \vec{v}_3 + (-\vec{v}_2)$



(i) (ii)

From diagrams (i) and (ii), the magnitude of acceleration is unchanged but direction is changing. This means the object is accelerating.

(b) (i) $\Delta v = (0.01)v$



(ii) Towards the centre of circle

(iii) Time $= \dfrac{\text{distance}}{\text{speed}} = \dfrac{r\theta}{v} = (0.01r/v)$ s

(iv) Acceleration $= \dfrac{\Delta v}{t} = \dfrac{0.01v}{\left(\dfrac{0.01r}{v}\right)} = \dfrac{v^2}{r}$

(c) (i) At top of loop, $N + W = \dfrac{mv^2}{r}$ .... (1)

Where N is contact force by track on carriages and W is weight of carriages.

$\Rightarrow$ For minimum speed, $N = 0$

$\therefore\ v = \sqrt{\dfrac{rW}{m}} = \sqrt{rg} = \sqrt{(8.6)(9.81)} = 9.2\ \text{ms}^{-1}$

(ii) Using (1): $N + W = \frac{mv^2}{r}$

$$\Rightarrow \quad N = \frac{mv^2}{r} - W = m\left(\frac{v^2}{r} - g\right)$$

$$= (800)\left(\frac{17^2}{8.6} - 9.81\right) = 1.9 \times 10^4 \text{ N}$$

(d) (i) Taking potential energy at ground to be zero.

Total energy at top of loop = Total energy at height x

$$mgh_1 + \frac{1}{2}mv^2 = mgh_2$$

$$(9.81)(2)(8.6) + \frac{1}{2}(17^2) = (9.81)x$$

$$x = 32 \text{ m}$$

(ii) There is no significant speed of carriages at height x.

(e) (i) Tension of string = Centripetal force

$$= \frac{mv^2}{r}$$

For the object moving at constant speed, the centripetal force increases as radius reduces. This will increase the tension in string and cause it to snap.

(ii) The attractive gravitational force on rings by planet Saturn causes the rings to centre on the planet.

Question 7

(a) Explain

(i) what is meant by a *radian*, [2]

(ii) why one complete revolution is equivalent to an angular displacement of 2π rad. [1]

(b) An elastic cord has an unextended length of 13.0 cm. One end of the cord is attached to a fixed point C. A small mass of weight 5.0 N is hung from the free end of the cord. The cord extends to a length of 14.8 cm, as shown in Fig. 1.1.

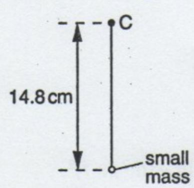


Fig. 1.1

The cord and mass are now made to rotate at constant angular speed ω in a vertical plane about point C. When the cord is vertical and above C, its length is the unextended length of 13.0 cm, as shown in Fig. 1.2.

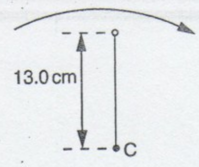


Fig. 1.2

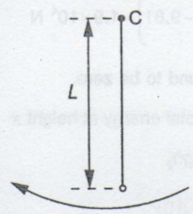


Fig. 1.3

(i) Show that the angular speed ω of the cord and mass is 8.7 rad s^{-1}. [2]

(ii) The cord and mass rotate so that the cord is vertically below C, as shown in Fig. 1.3.
Calculate the length L of the cord, assuming it obeys Hooke's law. [4]

[N07/P4/Q1]

Suggested Solution:

(a) (i) It is an angle subtended at the centre of circle by an arc equal in length to the radius of circle.

(ii) Since arc length $= r\theta$ and for one revolution, arc length $= 2\pi r$.

So $\theta = \frac{2\pi r}{r} \Rightarrow \theta = 2\pi$ rad.

(b) (i) Since tension in cord is zero, therefore

weight = centripetal force

$$mg = mr\omega^2$$
$$g = r\omega^2$$
$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{9.81}{13.0\times10^{-2}}} = 8.68 \text{ rads}^{-1}$$

(b) (i) Full explanation is required when the command word in a question is 'show' i.e. one must mention the source of centripetal force here to get full marks.

(ii)

$$T - mg = mr\omega^2$$
$$Ke = mg + mr\omega^2$$
$$\left(\frac{\omega}{x}\right)e = mg + mr\omega^2$$
$$\left(\frac{5.0}{(14.8-13)\times10^{-2}}\right)(L-13)\times10^{-2} = 5.0 + \left(\frac{5.0}{9.81}\right)(L\times10^{-2})(8.7)^2$$
$$\frac{5}{1.8}(L-13) = 5.0 + 38.6(L\times10^{-2})$$
$$\frac{5}{1.8}L - 36.1 = 5.0 + 0.386L$$
$$2.78L - 0.386L = 5.0 + 36.1$$
$$2.39L = 41.1$$
$$L = 17.2 \text{ cm}$$

Question 8

(a) **(i)** Define the *radian*. [2]

(ii) A small mass is attached to a string. The mass is rotating about a fixed point P at constant speed, as shown in Fig. 1.1.

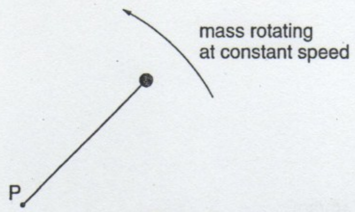


Fig. 1.1

Explain what is meant by the *angular* speed about point P of the mass. [2]

(b) A horizontal flat plate is free to rotate about a vertical axis through its centre, as shown in Fig. 1.2.

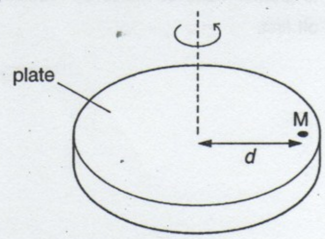


Fig. 1.2

A small mass M is placed on the plate, a distance d from the axis of rotation. The speed of rotation of the plate is gradually increased from zero until the mass is seen to slide off the plate.

The maximum frictional force F between the plate and the mass is given by the expression

$$F = 0.72W,$$

where W is the weight of the mass M.
The distance d is 35 cm.

Determine the maximum number of revolutions of the plate per minute for the mass M to remain on the plate. Explain your working. [5]

(c) The plate in **(b)** is covered, when stationary, with mud.

Suggest and explain whether mud near the edge of the plate or near the centre will first leave the plate as the angular speed of the plate is slowly increased. [2]

[J08/P4/Q1]

Suggested Solution:

(a) **(i)** It is an angle subtended at the centre of circle by an arc equal in length to the radius of circle.

(ii) Rate of change of angular displacement by the string.

(a) (ii) Angle swept out by the string due to its motion about P per unit time is the angular speed.

(b) The frictional force F provides required centripetal force

$$\therefore \quad F = mr\omega^2$$
$$0.72W = md\omega^2$$
$$0.72(mg) = md\omega^2$$
$$0.72g = d\omega^2$$
$$0.72(9.81) = (0.35)\omega^2$$
$$\omega = \sqrt{\frac{(0.72)(9.81)}{0.35}} = 4.49 \text{ rads}^{-1}$$
$$\Rightarrow \quad 2\pi f = 4.49$$
$$2\pi\left(\frac{n}{t}\right) = 4.49 \quad \Rightarrow \quad n = \frac{(4.49)(t)}{2\pi} = \frac{(4.49)(60)}{2(3.14)} = 42.9$$

$\therefore$ maximum number of revolutions = 43 min^{-1}

(b) One has to explain the initial situation and stages in their calculation to get full marks.

(c) Centripetal force at the edge is larger due to greater value of radius by relation $F = mr\omega^2$. Therefore mud at the edge flies off first.