I.G.C.S.E. Factorisation & Simultaneous Equations

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Question 1

Factorise the following expressions completely

a. $x^2 - 5x$ **b.** $7y^2 - 49y$ **c.** $3ab^2 + 6a^2b$ **d.** $xyz^2 - xy^2z + x^2yz$

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Solution to question 1

a. $x^2 - 5x = x(x-5)$

b. $7y^2 - 49y = 7y(y-7)$

c. $3ab^2 + 6a^2b = 3ab(b+2a)$

d. $xyz^2 - xy^2z + x^2yz = xyz(z-y+x)$

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Question 2

Factorise the following expressions

a. $ax + xz + ay + yz$

b. $2ap + 6aq - bp - 3bq$

c. $3uv - 9su - t^2v + 3st^2$

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Solution to question 2

Notice the two brackets are the same. Which now become a common factor

a. $ax + xz + ay + yz = x(a+z) + y(a+z)$

$= (a+z)(x+y)$

Notice the sign change

b. $2ap + 6aq - bp - 3bq = 2a(p+3q) - b(p+3q)$

$= (p+3q)(2a-b)$

Notice the sign change

c. $3uv - 9su - t^2v + 3st^2 = 3u(v-3s) - t^2(v-3s)$

$= (v-3s)(3u-t^2)$

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Question 3

Factorise the following quadratic expressions

a. $x^2+10x+24$ **b.** $x^2+2x-35$ **c.** $b^2-28b+75$

d. $k^2-29k-170$ **e.** $6f^2+17f+5$ **f.** $25y^2-20y+4$

g. $14z^2-19z-3$ **h.** a^2-9 **i.** $x^2-\frac{1}{4}$

j. $25y^2-49x^2$ **k.** $3x^3-27x$

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Solution to question 3

a. $x^2+10x+24$

product $=24$
sum $=10$
factors $=4,\ 6$

$\Rightarrow x^2+4x+6x+24$
$=x(x+4)+6(x+4)$
$=(x+4)(x+6)$

b. $x^2+2x-35$

product $=-35$
sum $=2$
factors $=-5,\ 7$

$\Rightarrow x^2-5x+7x+35$
$=x(x-5)+7(x-5)$
$=(x-5)(x+7)$

c. $b^2-28b+75$

product $=75$
sum $=-28$
factors $=-3,-25$

$\Rightarrow b^2-3b-25b+75$
$=b(b-3)-25(b-3)$
$=(b-3)(b-25)$

d. $k^2-29k-170$

product $=-170$
sum $=-29$
factors $=-34,5$

$\Rightarrow k^2-34k+5k-170$
$=k(k-34)+5(k-34)$
$=(k-34)(k+5)$

e. $6f^2+17f+5$

product $=30$
sum $=-17$
factors $=2,15$

$\Rightarrow 6f^2+2f+15f+5$
$=2f(3f+1)+5(3f+1)$
$=(3f+1)(2f+5)$

f. $25y^2-20y+4$

product $=100$
sum $=-20$
factors $=-10,-10$

$\Rightarrow 25y^2-10y-10y+4$
$=5y(5y-2)-2(5y-2)$
$=(5y-2)^2$

g. $14z^2-19z-3$

product $=-42$
sum $=-19$
factors $=2,-21$

$\Rightarrow 14z^2-21z+2z-3$
$=7z(2z-3)+(2z-3)$
$=(2z-3)(7z+1)$

h. $a^2-9=(a+3)(a-3)$

Using
$a^2-b^2=(a+b)(a-b)$
$a=a, b=3$

i. $x^2-\frac{1}{4}=\left(x+\frac{1}{2}\right)\left(x-\frac{1}{2}\right)$

j. $25y^2-49x^2=(5y+7x)(5y-7x)$

k. $3x^3-27x=3x\left(x^2-9\right)=3x(x+3)(x-3)$

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Question 4

Solve the following simultaneous equations

a. $4x+3y=2$
$3x-4y=14$

b. $5x+6y=5$
$2x+7y=2$

c. $3x-2y=-5$
$x-2y=1$

d. $\frac{x}{5}+\frac{y}{4}=-3$
$\frac{4x}{5}+\frac{7y}{8}=-2$

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Solution to question 4

a. $4x+3y=2 \quad \ldots 1. \xrightarrow{\times 4} 16x+12y=8$ Add

$3x-4y=14 \ldots 2. \xrightarrow{\times 3} 9x-12y=42$

$25x \quad = 50$

$x=2$

substitute $x=2$ into equation 1 we have $4(2)+3y=2$

$8+3y=2$

$3y=-6$

$y=-2$

$x=2, y=-2$

b. $5x+6y=5 \ldots 1. \xrightarrow{\times 2} 10x+12y=10$ Subtract

$2x+7y=2 \ldots 2. \xrightarrow{\times 5} 10x+35y=10$

$-23y=50$

$y=0$

substitute $y=0$ into equation 1 we have $5x+6(0)=5$

$5y=5$

$y=1$

$x=1, y=0$

c. $3x-2y=-5$

$x-2y=1$

Subtract

$2x \quad = -6$

$x=-3$

substitute $x=-3$ into equation 1 we have $3(-3)-2y=-5$

$-9-2y=-5$

$-2y=4$

$y=-2$

$x=-3, y=-2$

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d. First rearrange both equations into the form $ax + by = c$.

$$\frac{x}{5} + \frac{y}{4} = -3 \Rightarrow \quad \frac{4x + 5y}{20} = 3 \Rightarrow \quad 4x + 5y = 60$$

$$\frac{4x}{5} + \frac{7y}{8} = -2 \Rightarrow \frac{32x + 35y}{40} = -2 \Rightarrow 32x + 35y = -80$$

Now solving as before we have

$$4x + 5y = 60 \quad \ldots 1. \xrightarrow{\times 7} 28x + 35y = 420$$

$$32x + 35y = -80 \ldots 2. \longrightarrow \quad 32x + 35y = -80$$

Subtract

$$-4x \qquad = 500$$

$$x = -125$$

substitute $x = -125$ into equation 1 we have $4(-125) + 5y = 60$

$$-500 + 5y = 60$$

$$5y = 560$$

$$y = 112$$

$x = -125, y = 112$

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Question 5

José has fifty coins all of them either 2 soles or 5 soles coins. If he has 154 soles altogether, form two equations and solve them.

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Solution to question 5

José has fifty coins all of them either 2 soles or 5 soles coins. If he has 154 soles altogether, form two equations and solve them.

Let x be the number of 2 soles coins and y the number of 5 soles coins.

Now form two equations

The total of 2 soles coins and 5 soles coins must add up to 154 soles

$2x+5y=154 \ldots 1.$

$x+y=50 \ldots 2. \xrightarrow{\times 2} 2x+2y=100$

Subtract

$3y=54$

$y=18$

The number of 2 soles coins and 5 soles coins must add up to 50

substitute $y=18$ into equation 2 we have $x+2(9)=50$

$x+18=50$

$x=32$

José has 32 two soles coins and 18 five soles coins.

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Question 6

A ship can travel 20 knots with the current and 14 knots against it. Form two equations and find the speed of the current and the speed of the ship in still water.

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Solution to question 6

A ship can travel 20 knots with the current and 14 knots against it. Form two equations and find the speed of the current and the speed of the ship in still water.

Let the speed of the ship be x knots and the speed of the current be y knots.

Now form two equations

The ship can travel 20 knots with the current

$$x + y = 20 \ldots 1.$$

Add

$$x - y = 14 \ldots 2.$$

The ship can travel 14 knots against the current

$$2x = 34$$

$$x = 17$$

substitute $x = 17$ into equation 1 we have $17 + y = 20$

$$y = 3$$

The ship's speed is 17 knots and the current is 3 knots.

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